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THE PROBLEM OF MEASUREMENT DATA COMPLEXITY, FOR EXAMPLE OF THE GENERAL MODEL OF THE CENTRAL RESPIRATORY GENERATOR AND RECURRENT PLOTS ANALYSIS

The purpose of this modeling paper is to show and, in some extent, to explore the utility of one nonlinear tool, the recurrence plot, in assessing physiological systems and states, for example of the respiratory system. During investigations, the authors first explore the question of interrelations between regular and disordered processes, implying remarkable influences of noise on rhythmicity of the physiological systems, here: the respiratory system. Next, the signals acquired in the Bonhoeffer-van der Pol model of the central respiratory generator are the basis for data analysis; accordingly to Paydarfar *et al.*, computational studies of the equations (1), whose qualitative behaviour is representative of many excitable system, are able to show phase responses adequate to experimental findings in the animal, in the context of phase resetting of the central respiratory oscillator. The applied topological (qualitative) description and its quantitative measures of complexity document the important sensitivity to temporal variations of the data set compositions.

Keywords: complex systems and data, time-series, nonlinear dynamics, recurrence plots, respiratory system.

1. INTRODUCTION

The issue of complexity is a natural feature of the perceived reality and thereby of recorded measurement data, as the acquired signals are a manifestation of the properties and behaviour of the observed object. Complexity can concern both the structural configuration of the system as well as the character of the behaviour of its individual parts. These factors determine the insight into the state of the object obtained by application of the measurement procedure.

The complexity is an open question undertaken on the various planes. One of the conceptions consists in a fusion-like description of an object [1, 2, 3, 4]. Since the fusion originally takes place at the level of the object, each selective measurement

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activity needs to be composed with the other ones, e.g. by hardware configuration of the measurement set [5, 6]. A good example of this can be the polysomnographic attempts to describe the behaviour and state of the human body. Expressed in a defined structure, the architecture of sensor placement enables multichannel recording which reflects the temporary nature of its chosen subsystems [7, 8]. Apparently, in opposition to the reductionistic idea of description of reality, the acquired data still represents intermingling interactions between such independent parts or connected with the inherent structural hierarchy. In addition to this comes the problem of nature of the process, i.e. if it possesses a deterministic or stochastic, stationary or non-stationary character, modelled with lumped or distributed parameters. The question how such properties intermingle in the total output is still open. The next fusion-like thread is finally the answer to the question about the interference of information from different structural levels on a micro and macro scale. In the context of minimizing the measurement invasiveness, the choice between a provocative and spontaneous test is problematic .

A proper example of appearance of the problem of object and data complexity is the respiratory system and the well-known but still unsolved question of the sleep apnea syndrome (SAS) [9, 10]. On the other hand, a certain solution in the area of measurement of such complex characteristics can be the recurrence plots with the recurrence quantification analysis [11, 12].

In the paper, the authors do not investigate the solution of a specific measurement problem in the domain of physiological measurements, but outline the general rules for object description in the interpretation of a complex system and with exploitation of mathematical modelling, computer simulations and dedicated non-atomistic theoretical tools of data processing. The report opens a series of issues of cognitive and technical nature which stand for the present and will be the future subject of research. The interesting result of the work is showing the importance of noisy processes in total qualitative functioning of the whole system and next documenting the potential of the chosen theoretical tool of evaluation of the complex measurement data for the estimation of nontrivial conditions and behaviour for SAS.

2. MATERIALS AND METHODS

2.1. Model formulation

Two main kinds of pathology of sleep apnea were identified during functioning of the respiratory system. An obturative regime of apnea results from changes of tissue properties in the section of the upper airways, thus it relates to mechanical factors. On the other hand, pathological disturbances in rhythm (dysrhythmias) are observed in the breathing pattern of children and adults with neurological and cardiopulmonary diseases – the central apnea. Then, the second one is associated directly with the centre which regulates respiration – a central neural oscillator that produces rhythmic output to the respiratory muscles. The mechanisms responsible for genesis of respiratory dysrythmias, and in consequence leading to apneic behaviour, are still poorly understood. Especially interesting is the question of interference of random and non-random components between subsystems at the various structural levels and its consequence for the quality of functioning of the whole respiratory system. From a metrological point of view, an additional problem is the evaluation and control of these processes.

To introduce the aforementioned general idea, equations (1) were used. They are the generalization of the classical relaxational oscillator by van der Pol, called in [13] the Bonhoeffer-van der Pol model (BvP).

$$\begin{aligned} \frac{dx}{dt} &= c \cdot \left(y + x - x^3/3 \cdot d + z\right), \\ \frac{dy}{dt} &= -(x - a + by)/c, \end{aligned} \tag{1}$$

where: 1-2b/3 ¡ a¡ 1, 0 ¡ b¡ 1 and b¡ c^2. Unless stated otherwise, $d = 1$.

The BvP analogue was chosen because of its simple application and significant abilities as regards simulations of the respiratory system and irregularities of its work. A singular point can be located in this model on both sides of the attractor, which enables the simulation of apneas both during inspiration and expiration.

The structure (1) is a simple representative of a broad class of non-linear systems that exhibit excitability and oscillation. The phase plane of the BvP model depicts the evolution of the two variables of state (x, y). It illustrates the various states of excitability in physiological form, i.e. resting, active, and refractory phases. We implemented mathematical formulas in Matlab as follows. The solutions of the differential equations were estimated using the Euler-Maruyama method of integration. We tested the relation between the length of step of integration (Δt) and the precision of the solutions. Finally, in the paper we use the constant $\Delta t = 0.06$. Pseudorandom Gaussian noise with standard deviation η was incorporated into each iteration of x and y. In the present study z was systematically varied, and $a = 0.7$, $b = 0.8$, $c = 3.0$; excitability of the BvP system was altered by changing the z value, which allowed for investigation of a broad range of behaviour. The value of d controls the amplitude of oscillations and is indirectly correlated with the distance from the singular point to the attractor-cycle orbit (without a noise value). Initial conditions for the simulations were set to $x(0) = 0.5$ and $y(0) = 0.5$.

2.2. Recurrence plots analysis

Recurrence is a fundamental characteristic of many real dynamical systems. The formal technical concept of recurrences was introduced by Henri Poincaré [14], and this idea has been developed especially vividly when computers raised the numerical efficiency of system analysis. In this domain, chaos theory is a great representative of the new directions in recurrence and system synchronization research which have given unique theoretical and experimental remarks. The next step was the introduction by Eckmann *et al.* [15] of the method of recurrence plots (RPs) in 1987 to visualize the recurrences of dynamical systems. It constitutes measurement tools of two kinds: qualitative – by observations of the topological texture of the graphs and quantitative – by exact evaluation of the defined coefficients within the confines of recurrence quantification analysis (RQA). The construction of the plots is as follows.

Suppose we have a trajectory $\{\vec{x}\}_{i=1}^{N}$ of the system in its phase space (a single measured time series can be projected into multidimensional space by embedding procedures introduced by Takens [16]). The components of these vectors could be, e.g. the position and velocity of a pendulum or quantities such as temperature, expired/inspired gas pressure and airflow in the airways, brain potentials and many others for the respiration or human body. The development of the system is then described by a series of these vectors, representing a trajectory in an abstract mathematical space. Then, the corresponding RP is based on the following recurrence matrix:

$$R_{ij} = \begin{cases} 1: & \vec{x}_i \approx \vec{x}_j, \\ 0: & \vec{x}_i \not\approx \vec{x}_j, \end{cases} \quad i, j = 1, ..., N, \tag{2}$$

where N is the number of considered states and $\vec{x}_i \approx \vec{x}_j$ means equality up to an error (or distance) ε. Note that this ε is essential as systems often do not recur exactly to a formerly visited state but just approximately. Roughly speaking, the matrix compares the states of a system at times i and j. If the states are similar, this is indicated by a one in the matrix, i.e. $\mathbf{R}_{i,j} = 1$. If on the other hand the states are rather different, the corresponding entry in the matrix is $\mathbf{R}_{i,j} = 0$. So the matrix tells us when similar states of the underlying system occur [12]. The recurrence plot is obtained by plotting the recurrence matrix and using different colours for its binary entries, e.g. plotting a black dot at the coordinates (i, j), if $\mathbf{R}_{i,j} \equiv 1$, and a white dot, if $\mathbf{R}_{i,j} \equiv 0$. Since $\mathbf{R}_{i,i} \equiv 1|_{i=1}^{N}$ by definition, the RP has always a black main diagonal line. Furthermore, the RP is symmetric by definition with respect to the main diagonal, i.e. $\mathbf{R}_{i,j} \equiv \mathbf{R}_{j,i}$.

The most popular among the quantitative measures (RQA) of experimental data analysis by recurrent plots are:
- recurrence rate (*RR*),
- (percentage) determinism (*DET*),
- average diagonal line length (*L*),
- entropy (*ENTR*),
- laminarity (*LAM*),
- trapping time (*TT*),
- recurrence times of first time (T_1),
- recurrence times of second time (T_2).

Close definitions, characteristic properties and interpretations of the aforementioned factors can be found, e.g. in [12, 17].

More information and a detailed review of recurrence plots (including evolutionary studies, theoretical explanations and examples of application) was depicted by Marwan *et al.* in a complete report [12]. In the paper we have used the Matlab toolbox – CRP Toolbox, version 5.12 – dedicated to complex analysis of data on the basis of recurrent plot theory, prepared by Nonlinear Dynamics Group, University of Potsdam.

3. RESULTS

The various behaviours of the respiratory oscillator model have been simulated during the first section of the research. The analyses were conducted from the point of view of abilities to generate periodic trends and the conditions proper for respiratory dysrhythmia appearance. In Fig. 1, the time series y of the model output and the phase portrait (x, y) were shown as a function of parameter d. It follows that there is a relationship between the amplitude of natural oscillation of the respiratory oscillator and the amplitude of the disturbance. The (x, y) trajectories for $d = 0.8$ and $d = 0.9$ show that the periods of dysrhythmia are associated with the low amplitude of the orbits near the attractor-cycle orbit. These smaller orbits circumnavigate around the singular point of the system and at times there is true arrhythmia when the trajectory is so close to the singular point that the observed fluctuations are indistinguishable from noise. In the simulated BvP system, patterns similar to Fig. 1 (case $d = 0.8$ and $d = 0.9$) are readily generated if the singular point is close to the attractor-cycle and sufficient noise is added to the system. Below $d = 1$ the singularity point is stable and there is no oscillation. One explanation for the prorhythmic effects of increasing d is that by increasing the distance from the singular point to the attractor, the noise is less likely to displace the trajectory off the attractor-cycle toward a dysrhythmic locus around or at the singular point. Dysrhythmia is favoured if there exists a basin within which trajectories converge to a singular point, allowing perturbations of sufficient strength to displace the trajectory off the attractor-cycle to this basin. On the other hand, noise or other perturbations can be prorhythmic. For example, if the system is at the stable singular point, noise of sufficient magnitude can cause displacements outside the basin. So that the next step was studying the effects of noise (η) on BvP activities for different levels of excitability z. As can be seen in Fig. 2 – Fig. 4, in the oscillatory BvP system increasing noise leads to increases in the incidence and duration of dysrhythmia up to some level of noise. Further increase in noise above this "critical" level results in reduction in dysrhythmia. Fig. 4 depicts the fact that noise can convert a non-oscillatory system to one that exhibits spontaneous cycles.

The next stage was testing the usefulness of the theoretical tools – recurrent plots and recurrent quantification analysis – in assessment of the characteristics generated in the Bonhoeffer-van der Pol model. Recurrence plots (Fig. 5 – Fig. 10) were prepared for the time-series from Fig. 1 (d = [0.2, 0.5, 0.8, 0.9, 1.0, 1.2]) and next the quantitative indexes were calculated for such topological representations – Fig. 11 to Fig. 16. The length of the window for the analyses was set to n = 300 samples and the embedding dimension to 2. The obtained textures correspond with the standard patterns described in literature [11, 12]. Noisy perturbations are represented by a relatively homogenous non-patterned graphical profile (Fig. 5 and Fig. 6), and as there are more and more periodical sequences in the output signal y, the texture becomes more and more cyclic (Fig. 7 – Fig. 10). It is distinctive that the recurrent plots are able to distinguish not only between quite different qualitative processes but even between the two noisy data sets (Fig. 5, Fig. 6), which is expressed additionally by the quantitative indexes (Fig. 11, Fig. 12). According to definition and intuition, entropy of disordered processes is smaller than that of periodic ones; the percentage level of *RR* and the determinism measure – *DET*, similarly. These show, that subtle changes in a time series can be detected by pertinent variables that reflect the amount of rule-obeying structure in dynamic (percent determinism), the current state of the system (percent recurrence) and the description of the degree of complexity (entropy) of the recurrence plot. Sensitive to parametric changes in the BvP model are also the other factors of the recurrence quantification analysis. It fills in complete the legitimacy for future applications of the discussed theoretical tools – RP and RQA – in the case of real physiological trends, e.g. acquired during polisomnography (Fig. 17).

4. SUMMARY

Systems usually exhibit complex outputs because they possess multiple variables that are interconnected by nonlinear feedbacks. Ideally it would be sufficient to record all participant variables to draw a conclusion on the system of interest. Pragmatically, this is seldom feasible. However, by employing embedding techniques it is possible to reconstruct a constellation of surrogate variables from a single observed variable [18, 19], in our case the output of the model of the central respiratory generator.

A record of respiratory activity may include breath-to-breath variability of several types: random uncorrelated, random correlated, periodic and nonlinear deterministic. Much has been (and is being) written about analyses of variability in data [20, 21, 22, 23] and a summary could not be conclusive. In this paper we illustrate how recurrence plots can take single physiological measurements, project them into multidimensional space by embedding procedures, and identify time correlations (recurrences) that are not apparent in the one-dimensional time-series.

The investigated example of a computer experiment demonstrates what important influence on the total output of the physiological system (here: the respiratory system) noisy processes can display. Furthermore, it proposes the exploitation of advantages of the integrative measure of temporal variability – recurrent plots and recurrence quantification analysis – that reflects the total complexity (i.e. variability) in a biologic signal with less initial concern for its type. This new theoretical tool dedicated to experimental data analysis is a still-developed idea. Although there are the examples of its practical application [24, 25, 26], also in the area of the physiological systems [27, 28], it is still a technique with unemployed potential in the sense of ability to extract the information from complex objects and data set. Only single examples treating the use of RPs in respiratory investigations can be found in literature [29, 30]. It is worth to note that there are numerous variants of the classical RP, e.g. the cross recurrence plot (CRP), which make it possible to evaluate interrelations between the two different signals, subsystems. So, it broadens the domain of analysis with the issue of synchronization. In this sense, exploration of polysomnographic data complexity acquired during sleep (e.g. in the direction of detection, analysis or prediction of the sleep apnea syndrome) can be purposeful. It is expected that such signals contain the information on peculiar malfunctions accompanying this pathology. Further research in the area of mathematical modelling of complex systems (here: the respiratory system) can be also very fruitful. In the case of comprehension and diagnosis of sleep pathology it could facilitate the extraction of the proper symptoms and their sources, for example by applying sensitivity-like methodology.

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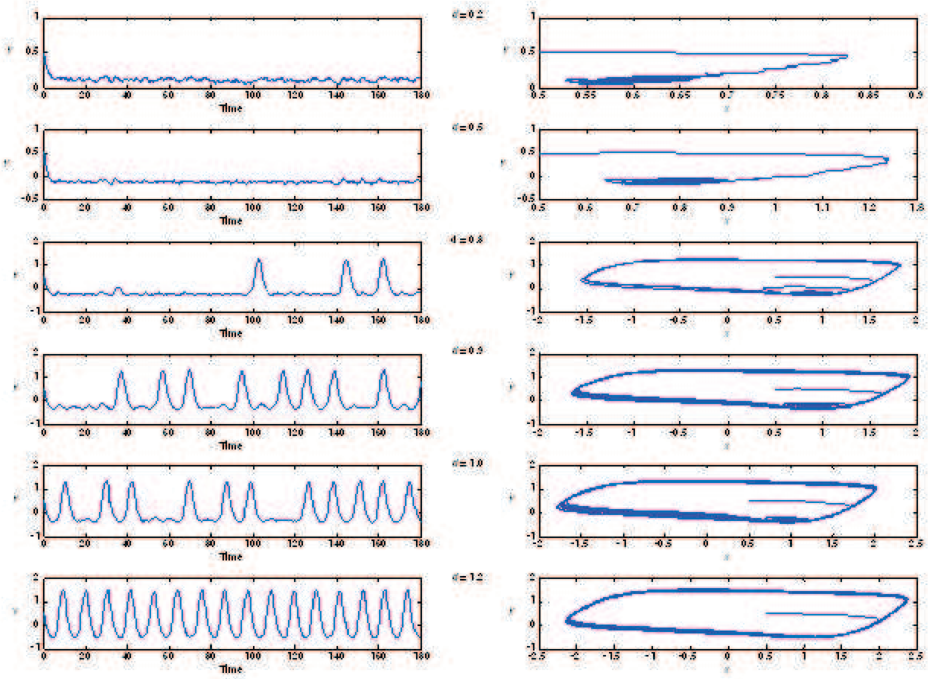


Fig. 1. Outputs $y(t)$ and phase portraits $(x,\ y)$ for the BvP model as a function of d; during all simulations z was set equal to –0.340.

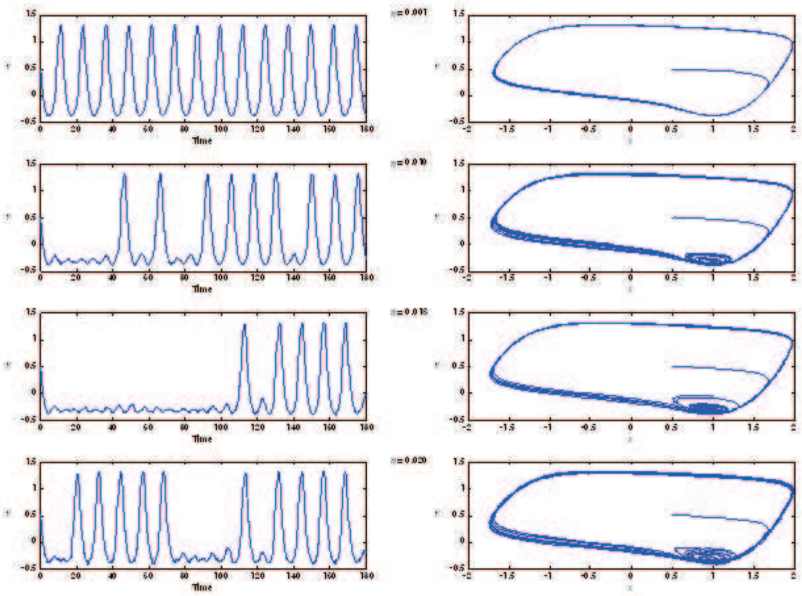
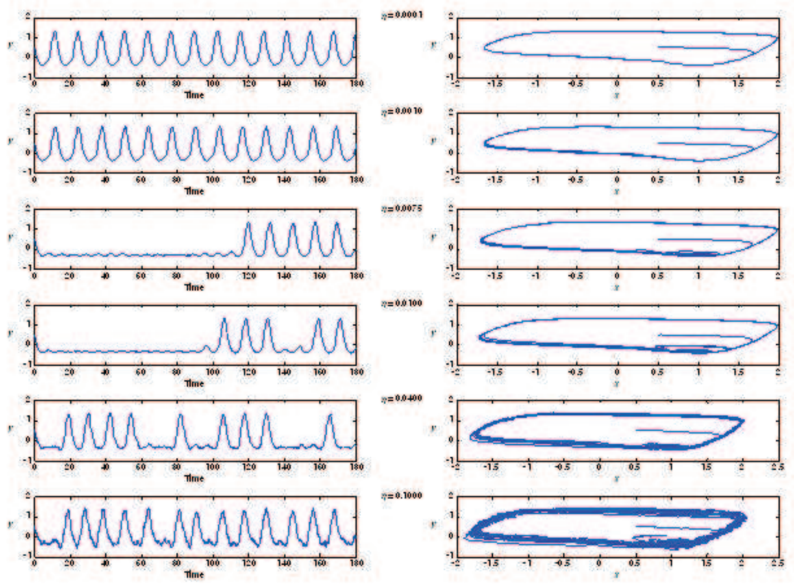


Fig. 2. Outputs $y(t)$ and phase portraits (x, y) for the BvP model for z = -0.335 and various levels of perturbations η.



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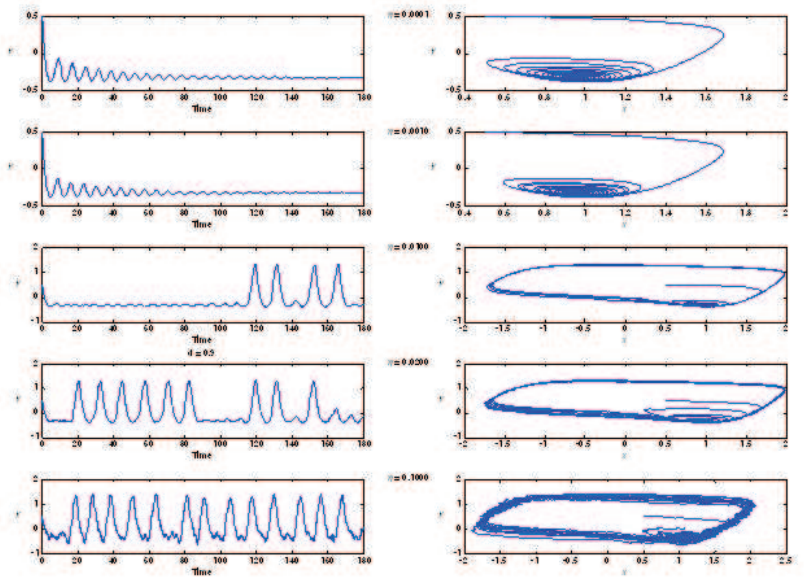

Fig. 4. Outputs $y(t)$ and phase portraits (x, y) for the BvP model for z = -0.345 and various levels of perturbations η.

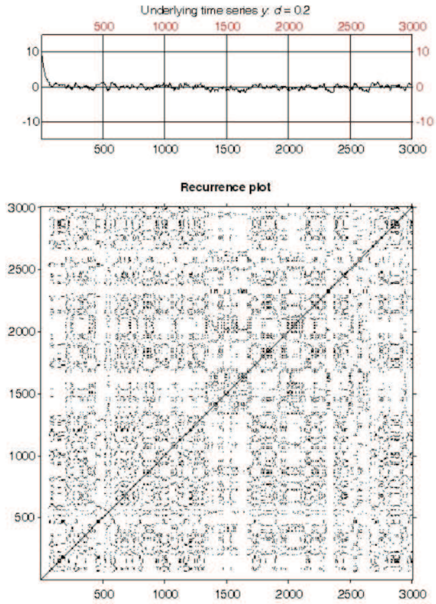

Fig. 5. Time-series $y(t)$ and recurrence plot for the BvP model; conditions as in Fig. 1 ($d = 0.2$).

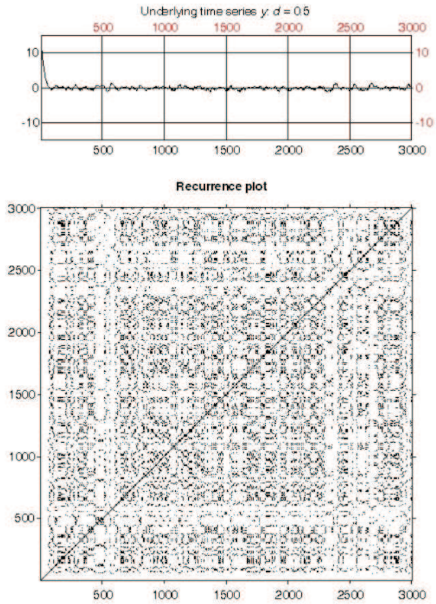

Fig. 6. Time-series $y(t)$ and recurrence plot for the BvP model; conditions as in Fig. 1 ($d = 0.5$).

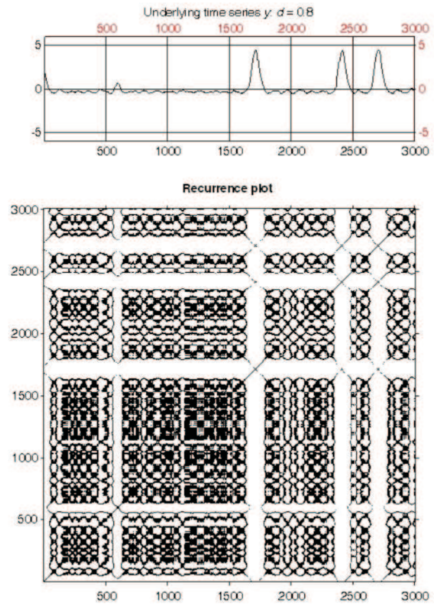

Fig. 7. Time-series $y(t)$ and recurrence plot for the BvP model; conditions as in Fig. 1 (d = 0.8).

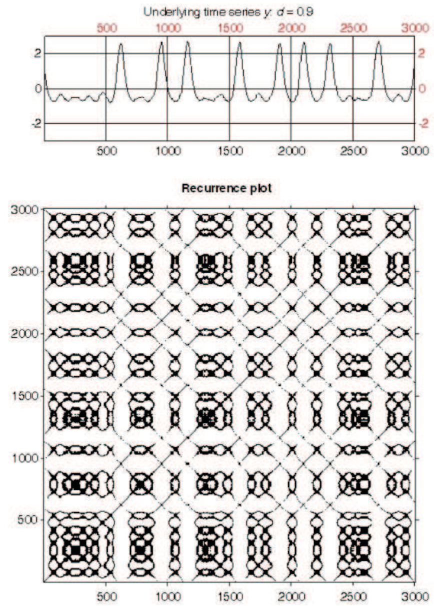

Fig. 8. Time-series $y(t)$ and recurrence plot for the BvP model; conditions as in Fig. 1 (d = 0.9).

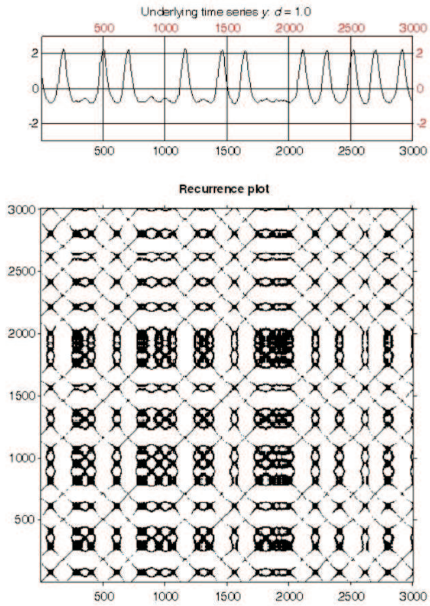

Fig. 9. Time-series $y(t)$ and recurrence plot for the BvP model; conditions as in Fig. 1 ($d = 1.0$).

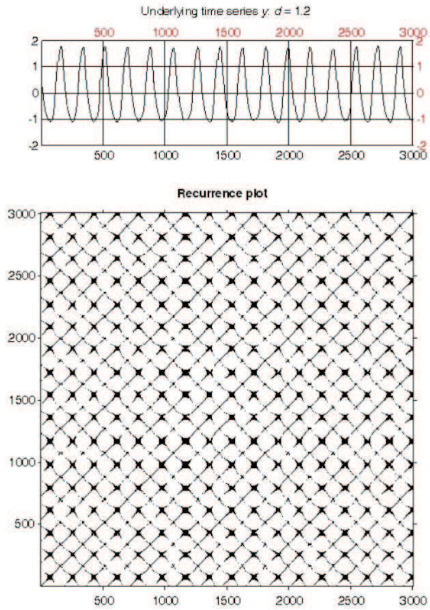

Fig. 10. Time-series $y(t)$ and recurrence plot for the BvP model; conditions as in Fig. 1 ($d = 1.2$).

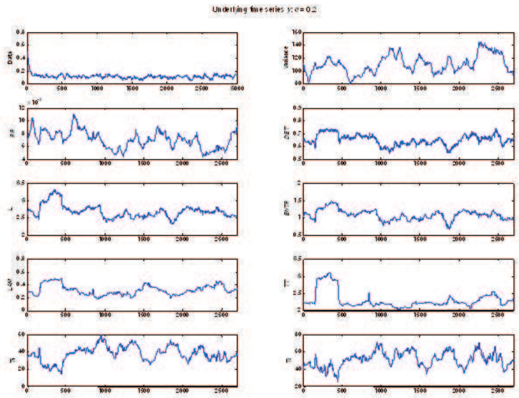

Fig. 11. Recurrent quantification analysis. Indexes calculated for the graph from Fig. 5.

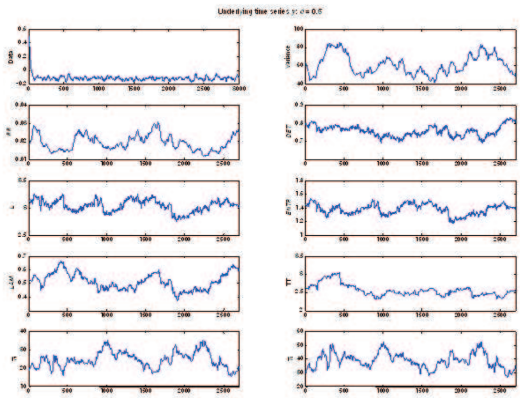

Fig. 12. Recurrent quantification analysis. Indexes calculated for the graph from Fig. 6.

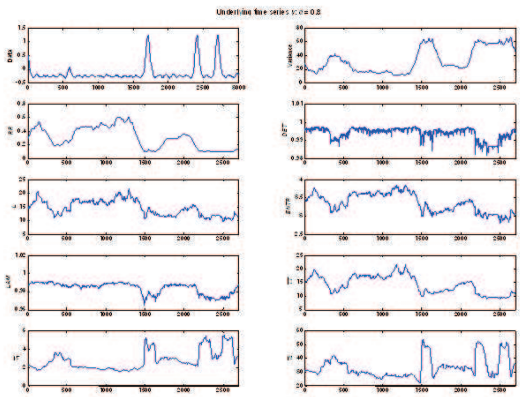

Fig. 13. Recurrent quantification analysis. Indexes calculated for the graph from Fig. 7.

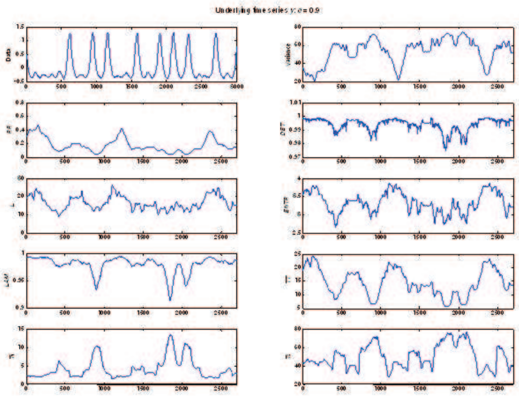

Fig. 14. Recurrent quantification analysis. Indexes calculated for the graph from Fig. 8.

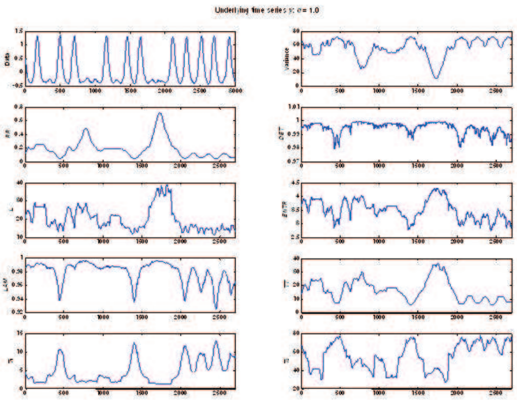

Fig. 15. Recurrent quantification analysis. Indexes calculated for the graph from Fig. 9.

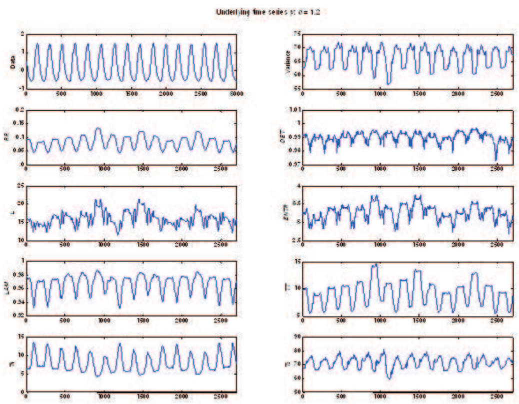

Fig. 16. Recurrent quantification analysis. Indexes calculated for the graph from Fig. 10.

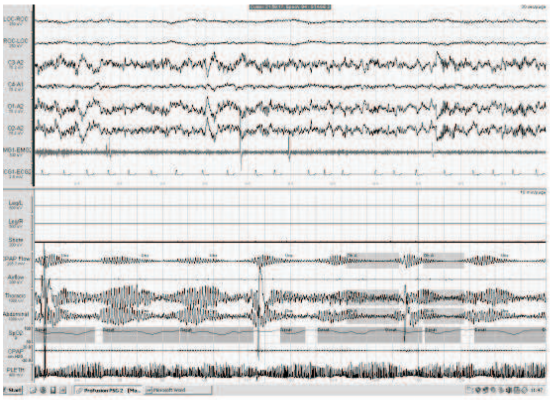

Fig. 17. Polisomnography data for patient with the symptoms of Cheyne-Stokes flow and accompanying sleep apnea syndrome.